# A generalized PEEC Analysis of Inductive Coupling Phenomena in a Transmission Line Right-of-Way

L. Blattner Martinho[1,2,3], J. Siau[2,3,4], B. Bannwarth[2,3], J.-M. Guichon[2,3], O. Chadebec[2,3], G. Meunier[2,3], V. C. Silva[1]

[1]Escola Politécnica da Universidade de São Paulo, LMAG, 05508-010 São Paulo, Brazil, blattner@usp.br
[2]Université Grenoble Alpes, G2Elab, F-38000 Grenoble, France, gerard.meunier@g2elab.grenoble-inp.fr
[3]CNRS, G2Elab, F-38000 Grenoble, France, olivier.chadebec@g2elab.grenoble-inp.fr
[4]CEDRAT, Meylan, France, jonathan.siau@cedrat.com

**A 3-D inductive coupling problem in a transmission line right-of-way is analysed with the generalized Partial Element Equivalent Circuit (PEEC) method. This new approach does not rely on the parallelism with the transmission line and permits the determination of the induced current density in underground objects at arbitrary positions and orientations.**

***Index Terms*—Eddy Currents, electromagnetic coupling, generalized partial element equivalent circuit (PEEC) method, integral equation, transmission lines.**

## I. Introduction

The inductive coupling between high voltage transmission lines and other structures sharing a common right-of-way is unavoidable. The numerical techniques used for the modelling of this class of problems frequently take advantage of the parallelism between the transmission line and elongated structures, as in the case of buried pipelines or transportation rails placed in a transmission line right-of-way [1] [2].

This paper analyses the modelling of an inductive coupling situation in which the referred hypothesis of parallelism between the transmission line and the object of interest is not required. The generalized Partial Element Equivalent Circuit (PEEC) integral method [3] is employed to determine the induced current density in an object beneath the soil surface and in the vicinities of a three-phase transmission line.

## II. PEEC Analysis of an Inductive Coupling

A schematic view of the right-of-way under analysis is available in Fig. 1. The buried object under investigation has a prismatic and elongated shape and is much more conductive than the surrounding soil, which is supposed to have a uniform and isotropic resistivity. The PEEC approach arises from an application of the Galerkin Residual method to the integral equation

$$\frac{\mathbf{J}}{\sigma} + j\omega \, \frac{\mu_0}{4\pi} \int_{\Omega} \frac{\mathbf{J}}{r} \, \mathrm{d}\Omega = -\nabla V, \tag{1}$$

which states the frequency domain relationship between the current density $\mathbf{J}$ and the electrical potential $V$ in a three dimensional domain $\Omega$ [3]. The media in $\Omega$ are supposed to be non-magnetic, non-dielectric and with conductivity $\sigma$. The procedure requires a finite element approximation for $\mathbf{J}$. Its particular choice and the assembly of the corresponding system of equations are detailed in the sequence.

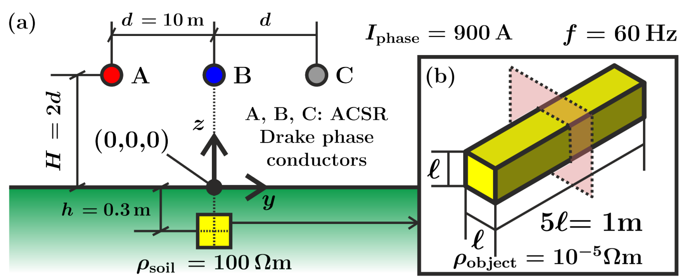


Fig. 1. The right-of-way (a) and the conductive object underground (b).

### A. Current density discretization and external circuit

Fig. 2 shows the delimitation of the domain $\Omega$ and its decomposition into two subregions. A sufficiently large soil volume $\Omega_{\mathrm{V}}$ bounded by the earth surface and containing the embedded underground object is defined. In $\Omega_{\mathrm{V}}$, $\mathbf{J}$ is approximated by vector facet elements [4]. The transmission line $\Omega_{\mathrm{L}}$ is represented by line elements, each one carrying a constant complex current and with one long element per phase conductor. This corresponds to the choice of zero-order interpolation functions with a pre-defined direction, limiting the number of additional degrees of freedom. After the Galerkin projection, this interpolation scheme yields the following system of equations:

$$(\,[R] + j\omega\,[L]\,)\,[I_f] = [\Delta V]\,, \tag{2}$$

in which $[R]$ and $[L]$ may be regarded as equivalent resistance and inductance matrices. This system gives an equivalent circuit interpretation of (1) in terms of both the facet and line currents in $[I_f]$, and as a consequence (2) may be coupled to an external network and analysed with an electrical circuit solver [3]. Fig. 2 also shows the required external circuit connections to provide current excitation and to establish an underground path for the flow of zero-sequence current components (if an unbalanced operation condition was to be considered).

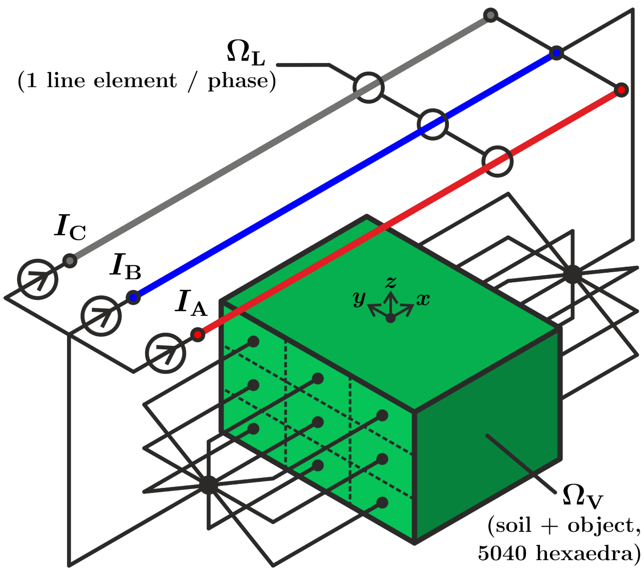


Fig. 2. The computational domain and its connections to the external circuit.

### B. Block integration and assembly of the system of equations

Matrices $[R]$ and $[L]$ have a 2X2 block structure due to the interpolation scheme adopted for $\mathbf{J}$. While $[R]$ is sparse, $[L]$ is densely populated and two of its blocks are dealt with in a particular way. The block corresponding to the $\Omega_{\mathrm{V}} \times \Omega_{\mathrm{V}}$ interaction is compressed using the $\mathcal{H}$-matrix representation and treated with the HCA technique [5]. The block associated to the $\Omega_{\mathrm{L}} \times \Omega_{\mathrm{L}}$ interaction is equivalent to the inductance matrix of the transmission line. This block is substituted by analytical computations of the conductor's self inductances and of the mutual inductances between phases. The two remaining blocks correspond to the inductive coupling between $\Omega_{\mathrm{L}}$ and $\Omega_{\mathrm{V}}$. Only one of these blocks needs to be computed since the other may be obtained by matrix transposition.

## III. Application and Results

Two relative positions between the line and the object are considered. For each one of them, the complete equivalent network arising from the numerical scheme has approximately 15000 branches and 9500 independent current loops.

First, the longest dimension of the object is placed in parallel to the line, in a configuration that permits the validation of the PEEC numerical scheme by comparison with the 2-D FEM solution of a related problem. In this latter approach the buried object is supposed to have an infinite length and $\mathbf{J}$ has a single component in the direction parallel to the transmission line. Due to a symmetry argument, this solution may be compared with the PEEC solution obtained in the mid-section of the buried object, which is highlighted in Fig. 1(b). A very good agreement between the two solutions is verified. For instance, if the mean current density is computed on a $0.25\delta \times 0.25\delta$ patch in the upper corner of the referred mid-section, an error lower than $2.15\%$ is verified between both solutions ($|\,\mathbf{J}_{\mathrm{PEEC}}\,| = 17.39\,\mathrm{A/m^2}$, $|\,\mathbf{J}_{\mathrm{FEM\,2\text{-}D}}\,| = 17.77\,\mathrm{A/m^2}$, $\delta = $ skin depth $\approx 0.205\,\mathrm{m}$).

In a second analysis, the object is rotated and positioned with its largest dimension along a direction orthogonal to the transmission line. The resulting configuration cannot be accurately modeled by a 2-D approach as previously. It can be nevertheless handled by the PEEC technique as before, and the current density distribution inside the object was once again determined by this procedure. Fig. 3 provides a sample of this distribution by showing the absolute value of each component of $\mathbf{J}$ on the horizontal mid-section of the object ($z = -0.3\,\mathrm{m}$).

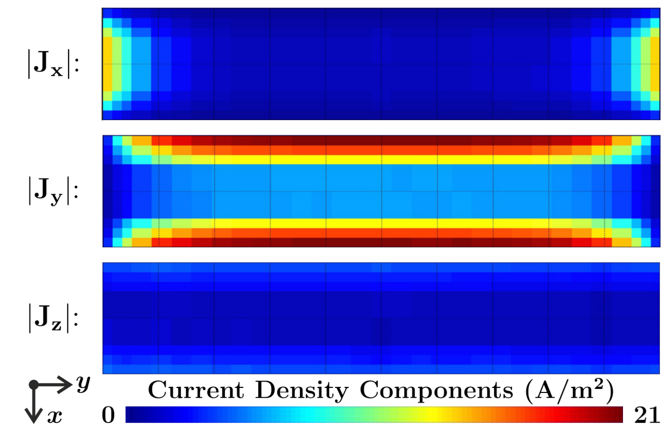


Fig. 3. Current density in the horizontal mid-section of the buried object.

## IV. Conclusion

A PEEC approach for the analysis of an inductive coupling problem involving a three-phase line and an underground object was presented. Arbitrary relative positions between the transmission line and the object could be considered, and the method also avoids the discretization of inactive air regions.

The current density distribution information obtained provides base data for the study of AC corrosion phenomena. Additionally, the electric potential solution obtained with the equivalent circuit interpretation may also be employed for the evaluation of dangerous induced overvoltages.

The previously discussed applications considered only the case of a balanced system of three-phase currents flowing in the phase conductors of the transmission line. Future work on this subject intend to investigate the case of unbalanced operation and the consequent superposition of conductive coupling phenomena, resulting from the flow of zero-sequence current components in the soil.

## References


[1] F. Dawalibi and R. Southey, "Analysis of electrical interference from power lines to gas pipelines. I. computation methods," *IEEE Trans. Power Del.*, vol. 4, no. 3, pp. 1840–1846, Jul 1989.

[2] G. Christoforidis, D. Labridis, and P. Dokopoulos, "Inductive interference on pipelines buried in multilayer soil due to magnetic fields from nearby faulted power lines," *IEEE Trans. Electromagn. Compat.*, vol. 47, no. 2, pp. 254–262, May 2005.

[3] T.-T. Nguyen, G. Meunier, J.-M. Guichon, O. Chadebec, and T.-S. Nguyen, "An integral formulation for the computation of 3-D eddy current using facet elements," *IEEE Trans. Magn.*, vol. 50, no. 2, pp. 549–552, Feb 2014.

[4] A. Bossavit, "Whitney forms: a class of finite elements for three-dimensional computations in electromagnetism," *IEE Proceedings Phys. Sci., Meas. Instrum., Manag. Educ. Rev.*, vol. 135, no. 8, pp. 493–500, November 1988.

[5] S. Börm and L. Grasedyck, "Hybrid cross approximation of integral operators," *Numerische Mathematik*, vol. 101, no. 2, pp. 221–249, 2005.